(Please write your Exam Roll No.) Exam Roll No. 09411903116 22

END TERM EXAMINATION

SECOND SEMESTER [B.TECH] JULY 2023

Paper Code: BS-106 **Subject: Engineering Mathematics-II**

Time: 3 Hours **Maximum Marks: 75**

Note: Attempt five questions in all including Q.No. 1 which is compulsory. Select one question from each unit. Use of non-programmable calculator is allowed.

Q1 Answer the following: [3x5=15]

a) Determine the value of P such that $f(z) = \frac{1}{2}\log_e(x^2+y^2) + i\tan^{-1}\frac{Px}{y}$ is an analytic function.

b) Find the image of $|z+1| = 1$ under the transformation $w = \frac{1}{z}$.

c) Find the Laplace transform of$(1+te^{-t})^3, t > 0$..

d) Determine the region in which $f(z) = (x-y)^2 + 2i(x+y)$ is analytic.

e) Write the general form of heat and wave equations in n dimensions.

UNIT-I

Q2 a) Given that any twice continuously differentiable function that satisfies the Laplace equation is harmonic, prove that $u(x,y) = x^2 - y^2 - 2xy - 2x + 3y, x, y \in \mathbb{R}$ is harmonic. Find a function $v(x,y)$ such that $f(z) = u + iv$ is analytic. [7]

b) Find the point and type of singularities of the following functions: [8]

(i) $f(z) = \frac{e^{\frac{1}{z}}}{(z-a)^2}$ (ii) $f(z) = \frac{1}{z^4+1}$

Q3 a) Find the Laurent's series of $f(z) = \frac{1}{z(1-z)}$ in region [10]

(i) $|z+1| < 1$ (ii) $|z+1| > 2$

b) Evaluate $\int_C \frac{z}{z^2-3z+2}dz$,where C is the circle $|z-2| = \frac{1}{2}$. [5]

UNIT-II

Q4 a) Show that the transformation $w = i\left(\frac{1-z}{1+z}\right)$ transforms the circle $|z| = 1$ onto the real axis of w-plane and interior of the circle $|z| < 1$ into the upper half of w-plane. [9]

b) Using the residue theorem, evaluate $\int_C \frac{e^z}{\cos(\pi z)}dz$, where C is the unit circle $|z| = 1$. [6]

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Q5 a) Find the residue of function $f(z) = \frac{1}{(z^2+1)^3}$ at $z = i$. [5]

b) Find the bilinear mapping that maps $-1, 0, 1$ of the z-plane onto $-1, -i, 1$ of the w-plane. Show that this maps the upper half of the z-plane onto the interior of the unit circle $|w| = 1$. [10]

UNIT-III

Q6 a) Find the inverse Laplace transform of $\frac{2s-5}{9s^2-25}$. [5]

b) Find the Fourier series expansion of the periodic function of order 2π [10]

$$f(x) = x^2, -\pi \leq x \leq \pi.$$

Hence, find the sum of the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$

Q7 a) Using Laplace transform, solve $y'' + y = \sin(3t), y(0) = 0, y'(0) = 0$. [7]

b) Find Fourier cosine transform of $f(x) = \begin{cases} x, & 0 < x < \frac{1}{2} \\ 1 - x, & \frac{1}{2} < x < 1 \\ 0, & x > 1. \end{cases}$ [8]

UNIT-IV

Q8 a) Derive the solution of the following wave equation using D' Alembert's method [9]

$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}, y(x, 0) = f(x)$ and $\frac{\partial y}{\partial t} = 0$ when $t = 0$.

b) Find the general solution of the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ using method of separation of variables. [6]

Q9 a) Use method of separation of variables to solve $\frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t}$, given that $v = 0$, when $t \to \infty$ and $v = 0$ at $x = 0$ and $x = l$. [9]

b) Given that $y(x,t) = \left(A\cos(ct\sqrt{k}) + B\sin(ct\sqrt{k})\right)\left(E\cos(x\sqrt{k}) + F\sin(x\sqrt{k})\right)$ is a general solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ where A, B, E, F, k are arbitrary constants. By putting the initial conditions $y = 5\cos(3t)$ when $x = l$ and $y = 0$ when $x = 0$, find the particular form of the solution. [6]